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A mathematical approach to determination of tax elasticity as a measure of direct tax productivity in Kenya

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Abstract
This research paper provides an eleven -year analysis of historical direct tax revenue performance and productivity in Kenya. The paper aims to give insights and inform budget making processes, planning and monitoring in Kenya by ensuring a balanced budget through productive tax yields. This paper is grounded on the basis that a good fiscal policy structures and system ensures stable government revenues through fair and sound taxation over time, improves efficiency and equity of taxes while promoting investment towards accumulation of wealth/national income and economic growth. Government of Kenya has continuously pursued tax reform agenda to increase its domestic's tax revenues mobilization. This study examined the elasticity of direct income taxes as a measure of tax productivity in Kenya. Specifically, the paper examined the elasticity of Corporation tax, Property taxes (rental taxes) and personal income taxes as components of direct government tax revenues with respect to the changes in national income/ GDP at factor income as a proxy base. The study population for the research was the years from 2007 to 2017 financial year spanning for a period of 11 years. The study relied on secondary data and employed a time series approach and techniques to estimate tax elasticity for various component of direct income taxes in Kenya for a period ranging from 2006/2007 to 2016/2017. Proportional Adjustment Method (PAM) model and Error Correction Mechanism (ECM) were adopted for data analysing. To check stationarity of the time series data, Phillips Perron (PP) test and Augmented-Fuller (ADF) test for unit root test techniques were adopted. The results revealed that direct taxes in Kenya are inelastic with elasticity value of 0.592 less than unit with an error correction coefficient of 0.7778. The study established that the direct tax revenue in Kenya is actually not responsive enough to changes in income growth since the coefficient of elasticity was less than a unity. Thus the system of direct taxes is not productive in general. Based on the findings of this paper, we strongly recommend that the government of Kenya should strengthen tax reforms in order to increase the productivity of income tax revenue by broadening and expanding the tax base in Kenya.

Keywords: Tax elasticity, direct tax productivity

Introduction
The public expenditures in Kenya have been increasing with a ballooning national debt surpassing KES 5 trillion while at the same time revenue generation continuously missing targets. Thus the question of how productive is the system of direct tax revenue in mobilizing domestic revenue. As Kenya's budget continues to grow in trillions of shillings, there is need to put up a healthy and productive fiscal management system that ensures stable revenues over time, improves equity and efficiency of taxes and promotes investment towards economic growth and increased national income.

The sheer size of that budget has occasioned discussions regarding the country's capability to raise the revenue to fund the expenditure plans given the level of the national debt book and; previous performances and overall budget absorption capacity at both levels of government. According to the Budget Policy Statement (BPS) 2018, the National Treasury reported the total cumulative revenue, including Appropriations in Aid, amounting to KES 1.48 trillion or an average Sh123.93 billion a month in the year ending June 2018 against a target of KES 1.65 trillion, implying a shortfall of KES 170 billion. The shortfall has lead policy makers to question the approach used in forecasting national revenue to finance the annual budget. This notwithstanding, Government's ambition to roll out a significant number

of infrastructural development has been poised to compound the budget financing challenges.

A tax is non penal transfer of resources from the citizen to the state made without reference to a specific benefit received from the government as a quid-pro-quo (Mutua, 2012; OECD, 2013) [15, 17]. In order to maintain provision of social services and adequate level of public investments, the governments require steady flow of domestic tax revenue to deter excessive external and internal government borrowing (Poulson & Kaplan, 2008) [19]. The necessity to raise and accumulate adequate domestic revenue against the backdrop of high public expenditure in Kenya has been the primary motivation for tax base reforms with aim of diversifying tax base and increased (Muriithi and Moye, 2003) [14].

Tax elasticity measures the responsiveness of tax revenue yields to a change in national income/output (nominal Gross Domestic Product (GDP)) or movements in economic activity after correction for any changes in the tax rates, tax base and the tax structure (Timsina, 2008) [26]. The concepts of elasticity in tax administration helps in determination of productivity of the tax structure and system. The revenue performance of any country is considered satisfactory enough on any given measurement scale if all the revenue sources available provide an upward trend in tax revenue per year and revenue sources are income elastic with reference to their proxy bases while generating commensurate tax revenues to finance the government's public expenditure. The importance of tax productivity cannot be negated because tax productivity allows prudent examination of the responsiveness of the tax system and this affects the tax system's efficiency and equity (Amin, 2000) [1]. Changes in tax base and tax rate, changes in national income, changes in tax assessment and collection efficiency, among others can affect tax productivity of a country. The productivity of the income tax revenue can be measured by measuring the response of income tax revenue to changes in national income or GDP as the proxy base. This can be done with the help of tax elasticity as a product of tax to base elasticity and tax to rate elasticity. Tax to base elasticity refers to the responsiveness of changes of proxy tax bases with respect to aggregate national income while tax to rate elasticity measures the responsiveness of the tax yield in respect to tax base (Murithi and Moyi, 2003; Moyi & Ronge, 2006; Samuel and Isaac, 2012; Omondi *et al.*, 2014) [14, 12, 25, 18]. Thus tax elasticity is a reconstruction of what would have happened if there had been zero changes in the tax structures or the tax rules (Mawia and Nzomol, 2013) [10].

The Kenya government has undertaken various tax reforms to accumulate more revenue to sustain the public finance expenditure with current tax base expansion an ongoing tax reforms. However, in spite of the ongoing reforms, tax revenue has been continuously falling below targets thus the need to investigate the productivity of the tax system through tax elasticity. Given that Kenya's revenue portfolio is highly driven by tax revenues, we pose to ask in this study whether we have established an optimal revenue structure for maximum yield, a tax system which is more elastict?

Literature Review

Often it is a valid expectation of a good sound tax system to generate and raise enough revenue to the government and is responsive to changes in the country's national income without introducing economic imbalances (Belinga *et al.*, 2014) [2]. Thus economies with efficient and effective tax system attempts to generate sufficient tax revenue for public finance expenditure without resulting to excessive borrowing or introducing economic imblalnces (Moyi & Ronge, 2006) [12]. This study compares tax revenue generation capacity and productivity to the portfolio theory of risk diversification. According to the portfolio theory, diversification reduces variability or risk as long as changes in various stock prices are not perfectly correlated or changes in different stocks does not go in the same direction (Ross, Westerfield & Jordan, 2008) [24]. Risk diversification helps eliminate unique risks as a result of adverse conditions surrounding a particular industry or company, however does not eliminate the market risk because it involves wide perils in the economy that affect all businesses (Ross, Westerfield & Jordan, 2008) [24].

In the field of taxation, the concept of diversification of tax revenue is analogous to investment diversification. Diversification of tax revenue sources can either be a strategic policy or a deliberate action aimed at widening the tax base to provide for flexibility and stability in financial management, in order to improve fiscal performance (Belinga *et al.*, 2014) [2], thus various tax bases or revenue sources are considered as investment portfolio of the government while each tax is viewed as a security in the portfolio. Tax revenue variability is similar to market returns volatility concept in corporate finance (Yan, 2008). Tax revenue diversification in the field of taxation is largely dependent on the tax revenues' income elasticity and is related to the coefficient of correlation between various taxes. A good tax structure should comprise taxes that do not have perfect correlation with each other so that fluctuation in revenue is reduced. In such a case, whenever revenue from one tax shrinks, the overall revenue loss to the government is minimized because similar changes have not been experienced in other sources of revenue (White, 1983).

Each tax's income elasticity depicts that different tax revenues have diverse degrees of sensitivity to the general conditions in the economy. Individual tax revenue's income elasticity is compared to the market risk of each security in the case of investment portfolio. According to Merriman & Dye (2004) [11], it is assumed that a revenue system that is inelastic will generally lead to a revenue system that is cyclically stable. Therefore, if the tax structure is modified such that it includes low elastic taxes, then there could be a reduction in revenue risk associated with economic cycles. The trade-off however is that during periods of economic boom, there will be minimal revenue growth.

In Pakistan, Mukariam (2001) [13] investigated tax elasticity and tax buoyancy of the Pakistan major taxes for the period from 1981-2001. His investigation used chain indexing technique to adjust the tax yield series to eliminate the revenue discretionary effect. Chain indexing technique was used to ensure that the tax revenues obtained for each year under investigation was due to the constant rates applicable in the year had prevailed throughout the reference year. Mukariam (2001) [13] used ordinary least squares (OLS) regression techniques for analysis and the results showed that, customs and domestic taxes were relatively rigid for the period under investigation.

In another study in Nepal, Timsina (2008) [26] sort to estimate the elasticity and the buoyancy of the tax system in Nepal. Timsina (2008) [26] used annual time series on tax revenue data for the period from 1975 - 2005. The study

used partitioning approach to estimate and measure the elasticity coefficients for the different taxes in Nepal. Thus the study estimated tax elasticity by estimating tax to base elasticity and base to income elasticity. The tax to base elasticity was used to measure the progressiveness of the tax system and structure in Nepal against the changes in administrative effectiveness and efficiency. The base to income elasticity was used to measure the responsiveness of tax base to changes in national income. The results showed that the Nepal tax system and structure was income inelastic with a coefficient of elasticity less than a unity in the period under investigation. This shows that the bulk of tax collection was due to discretionary measures in the Nepal tax policy and not from system built up automatic responses.

In Kenya, Omondi *et al.* (2014) [18], explored the effectiveness of tax reforms on the tax system elasticity of the with regards to the revenue administration reforms and tax modernization programme in Kenya. The study used time series data on annual basis from the period 1963 to 2010 to estimates elasticity and tax buoyancy and to determine the effect of the revenue administration reforms and tax modernization programme. The study used regression model and regressed that tax revenue on national income in Kenya. Their findings showed an elasticity coefficient of 0.690 for the overall tax system in Kenya. This implied that the responsiveness to changes in national income brought about a less proportionate changes in the tax revenue in Kenya during the period of study. This findings conformed to the other previous findings such as the findings by Wawire (2006) [29] who asserted that the tax system in Kenya was income inelastic during the period under investigation.

Research Methodology

This study employed a causal research techniques to determine how one variable causes or is responsible for the changes in other variables (Cooper and Schindler, 2006) [3]. An extensive desk assessment to ascertain the gaps in research was conducted to inform the objectives of the study. This referred to the on desk review that analyzed various reports from scholars, government entities and departments on the subject matter. A multiplicative model was employed on the historical time series revenue data, collected from Kenya Revenue Authority Abstracts, Kenya Statistical Abstracts and World Bank economic reports. This was to adjust the time series revenue data for irregularity and seasonality in the secondary data. The multiplicative model is normally employed where seasonal data variation increases over time as witnessed in the annual revenue data (Purohit, 2005) [21]. The time series revenue data of various tax heads, total revenue and the GDP at factor income for 11 years from 2006 to 2017, was collected from the published economic reports.

The analysis used a Tax Elasticity Model to estimate the response of the direct income taxes to changes in gross national income, in order to measure tax productivity of Kenya's tax system. Although this approach produces a more reliable results, it was very data demanding in terms of the requirement of the specific information of the discretionary measures and policy that were applicable during the period under investigation.

In identifying the appropriate proxy tax base for the elasticity model, the study used GDP at factor income as the tax base to establish the elasticity on all of the direct taxes involved. However the research recognizes that other tax bases such as production, consumption and national income can enhance the reliability of the findings and the results, the quarterly tax revenue from 2006/2007 to 2016/2017 was unavailable thus the researchers chose to utilize GDP at factor as the most appropriate proxy tax base. Thus the analysis utilizes the Corporation tax revenue data, Personal income tax revenue data, Property tax (rental tax) revenue data and the data on gross GDP at factor income to establish tax elasticity as a measure of the productivity of the tax system in Kenya.

Tax Elasticity estimation

Tax elasticity estimates the tax revenue response to a change in national income/output (nominal Gross Domestic Product (GDP)) or movements in economic activity but after correction for any changes in the tax rates, tax base or the tax structure (Timsina, 2008) [26]. Unlike tax buoyancy, the estimation of tax elasticity requires that discretionary measures be controlled or adjusted from the actual revenue. However, when the instruments of tax policy are subjected to drastic changes from time to time, the estimation of tax elasticity with considerable accuracy may be difficult.

Elasticity approach relies on the completeness of the necessary information to adjust and isolate discretionary policies thus provide the necessary information to understand the responsiveness of tax system. Thus estimation of elasticity of tax system requires an adjustment of the revenue data from the actual revenue yield so as to isolate the revenue growth arising from automatic changes due to discretionary measures (Belinga *et al.*, 2014) [2].

The available empirical literature provides for four mathematical approaches to estimating tax elasticity such as; the constant rate structure, proportional adjustment method (PAM) and divisia index approach. However, all these approaches outlined depend heavily on the nature, availability and reliability tax revenues data. The Proportional Adjustment Method (PAM) was used to adjust data to eliminate a series of discretionary changes so as to give a clear estimates income tax elasticity. This method separates data on revenue brought due to changes in discretionary measures based on the government data so as to get a precise reflection of tax revenue actually collected if the structure of tax systems, rate and base had been applicable throughout the sample period (Choudhry, 1979; Haughton, 1998) [4, 7].

This study used a mathematical regression approach to estimate and measure the elasticity of the direct income taxes in Kenya. The mathematical equations used was as follows:

Equation 1: Tax Elasticity

$$TR=\alpha Y^{\beta i}\varepsilon \quad \text{..... (i)}$$

Where
TR = Direct income tax revenue,
Y= GDP at factor income at time *i* and
β_i = Coefficient of elasticity of the i^{th} tax.

To transform the mathematical equation (i) to linear equations we apply the concept of the natural logarithims on both side of the equation thus;

Equation 2: Log of Tax Elasticity

$LogTR_i = Log\alpha + \beta_i LogY + log\varepsilon_i$…… (ii)

Resulting to a standard format;

$LogTR_i = \alpha + \beta_i LogY + \varepsilon_i$…….. (ii)

Where;
β_i; is the coefficient of elasticity that shows the percentage change in tax revenue a result of 1% change in income.
Thus the mathematical regression model regresses the log of direct income tax revenue on the log of the GDP as the proxy base. The coefficient on the log of GDP is then interpreted to be a measure of the tax elasticity.
To eliminate the discretionary effects, other econometric methods were used for adjustment of time series revenue data to eliminate for discretionary effects of the tax policy. This econometric methods was used as equation 3.

Equation 3: Elimination of discretionary effects

$TR_i = T_i - D_i$

Where:
T_i = Actual yield in direct tax revenue in the i^{th} year
D_i = Budgetary estimate catering for the discretionary change(s) in the i^{th} year
TR_i = Adjusted actual direct tax revenue collection of the i^{th} year.

Because data used was time series data, it was most likely that there would be presence of unit roots in the time-series of various tax revenues for the 11 years from 2006 to 2017. Other econometric methods were used for stationarity test that were conducted to ensure the data does not give spurious results due to non-constant mean and variance. Thus;

1. Augmented Dickey- Fuller (ADF) and Phillips Perron (PP) test unit root were employed to determine whether or not the data is stationary.
2. Sampled data was examined and partial autocorrelations carried out on the time-series data.
3. To check for long-run relationship, Johansen Cointegration tests was done. The regression residuals were integrated of order zero [1(0)] or stationary.
4. Granger causality test was carried to establish the causation between tax revenue and GDP in Kenya.

Definition of Variables

Table: Table of variables

Variable	Definition and Measurement
Direct Tax Revenue (T)	This is the tax revenues that are borne entirely by the individual/or entity that pays it, and cannot be passed on to another entity.
Gross Domestic Product (Y)	This refers to the aggregate value of all goods and services measured in Kenya shillings that are produced in a country in a period of one year that are produced by both foreigners or residents.

Results and Discussions

Unit root tests

Unit root tests for all variables were performed using ADF test and PP test. The null hypothesis and the alternative hypothesis for stationary test were taken to be:
H0: there is a unit root and the times series is not stationary
HA: There is no a unit root and the times series is stationary
The null hypothesis can only be rejected when the absolute value of the test statistic ought to have been greater than the absolute critical value at a given percentage level of significance. The results for the Augmented Dickey- Fuller (ADF) test are presented in table 1 below.

Table 1: ADF unit root test for variables in levels

Variables	Trend	Lags	AD F	Probability	Decision	
Log of direct taxes	Yes	0	-1.872	0.629	Non-Stationary	
Log Corporation Income Taxes	Yes	0	-.052	0.144	Non- Stationary	
Log Personal Income Taxes	Ye s	0	-.42 6	0.977	Non- Stationary	
Log Rental income Taxes	Ye s	0	-4.201	0.018**	Non-Stationary	
Log of GD P	Ye s	0	-1.708	0.707	Non-	Stationary
Lo g of GD P at factor income	Yes	0	-2.394	0.370	Non-	Stationary
Log of total Taxes	Yes	0	-2.184	0.470	Non-	Stationary
Log of adjusted direct Taxes	N o	0	-0.08 9	0.937	Non-	Stationary
Log of adjusted total taxes	Ye s	0	-2.07 5	0.525	Non-	Stationary

- ** mean s stationary at 5% level
- Time variable: Year, 2007 to 2017

Table 1 shows that all the variables were non-stationary in their respective levels. The Augmented Dickey- Fuller test results indicate that all of the variables transformed in logarithms are not stationary in their levels. Thus the presence of unit root can be rejected at 5% level basing on MacKinnon probability values as shown in table 2 below.

Table 2: Interpolated Augmented Dickey-Fuller (ADF)

Test Statistic		Critical Value of 1%	Critical Value of 5%	Critical Value of 10%
Z_t	-2.717	-3.580	-3.019	-2.720

Approximate MacKinnon Z_t p-value = 0.0412
Time variable: Year, 2007 to 2017

The result of the Interpolated Augmented Dickey-Fuller (ADF) as shown in table 2 above shows that the presence of the unit root was rejected both at 1% significant level and 5% significant level basing on MacKinnon probability values. The Interpolated Augmented Dickey-Fuller (ADF) coefficients are shown in table 3 below.

Table 3: Interpolated Augmented Dickey-Fuller (ADF) Coefficients

D.DTR	Coef.	Std. Err.	T	P>|t|	[95% Conf. Interval]	
					Lower limit	Upper Limit
DTR	-.9497447	.3495735	-2.72	0.026	-1.755863	-.1436269
_cons	-74739.83	57519.85	-1.30	0.230	-207380.8	57901.18

Time variable: Year, 2007 to 2017

To confirm these results, the PP test was performed and results displayed in table 4 above. Thus the data is non-stationary time series.

Table 4: PP unit root test for variables in levels.

Variables	Trend	Lags	PP	Probability	Decision	
Log of Corporation Income Taxes	Yes	0	-1.952	0.588	Non- Stationary	
Log Corporation Income Taxes	Yes	0	-.0520	0.144	Non- Stationary	
Log Personal Income Taxes	Ye s	0	-4.86 8	0.005***	Non-Stationary	
Log Rental income Taxes	Ye s	0	-.22 80	0.44 8	Non- Stationary	
Log of GD P	Ye s	0	-1.70 8	0.707	No n-	Stationary
Lo g of GD Pat factor income	Yes	0	-2.394	0.37	Non-	Stationary
Log of Personal Income Taxes	Yes	0	-2.22 8	0.44 8	No n-	Stationary
Log of adjusted Rental Income Taxes	N o	0	-0.13 6	0.931	No n-	Stationary
Log of adjusted Total Direct Taxes	Ye s	0	-2.141	0.492	Non-	Stationary

- *** mean s stationary at 1% level
- Time variable: Year, 2007 to 2017

The PP test results confirmed the ADF test results obtained above. Thus it can be concluded that the presence of unit root was rejected at 1% level based on MacKinnon p-values. However, the presence of unit root cannot be rejected at 10% level for all the variables.

Table 5: Interpolated Phillips-Perron (PP)

Test Statistic		Critical Value of 1%	Critical Value of 5%	Critical Value of 10%
Z_{rho}	-0.512	-16.200	-12.400	-11.201
Zt	-0.278	-4.651	-3.156	-2.820

Approximate MacKinnon $Z_{(t)}$ P-value = 0.9286
Time variable: Year, 2007 to 2017

Form the above, the presence of non-stationary variables in all equations to be estimated compels the researcher to use appropriate econometrics technique to deal with that problem. In that sense, the cointegration technique developed by Engle and Granger (1987) was employed in this study. This technique helped us to avoid the problem of spurious regression and to derive the long and short run relationship between variables via error correction modeling.

To check for the order of integration of non-stationary variables, the latter were differenced once and ADF and PP tests were performed again. The same procedure was followed in choosing the optimal lag length and a constant was also included while no trend was allowed in the regressions. The results of the ADF test and PP test were reported in table 6 below.

Table 6: ADF unit root for variables in difference

Variables	Trend	Lags	AD F	Probability	Decision	
Log of direct taxes	Yes	0	-1.394	0.025**	Stationary 1(1)	
Log Corporation Income Taxes	Yes	0	-3.394	0.025	Non- Stationary I(2)	
Log Personal Income Taxes	Ye s	0	-3.581	0.017	Stationary I(2)	
Log Rental income Taxes	Ye s	0	-4.101	0.028	Stationary I(2)	
Log of GD P	Ye s	0	-4.105	0.006***		Stationary I(2)
Lo g of GD P at factor income	Yes	0	-4.637	0.002* **		Stationary 1(1)
Log of total Taxes	Yes	0	-2.184	0.470		Stationary 1(1)
Log of adjusted direct Taxes	N o	0	-3.640	0.015	Non-	Stationary I(2)
Log of adjusted total taxes	Ye s	0	-4.562	0.002 * **		Stationary 1(2)

- *** means stationary at 1% level
- I(1) means integrated of order one
- 1(2) mean s integrated of order two
- Time variable: Year, 2007 to 2017.

Basing on Mackinnon p values, the null hypothesis of presence of unit root was rejected for all variables in their first difference except the except for log of GDP. To validate these results, the PP tests gave us the following results in table 7 below.

Table 7: PP unit root for variables in difference.

Variables	Trend	Lags	PP	Probability	Decision
Log of total direct taxes	Yes	0	-3.648	0.015**	Stationary 1(1)
Log Corporation Income Taxes	Yes	0	-3.385	0.025**	Non- Stationary I(2)
Log Personal Income Taxes	Ye s	0	-4.86 8	0.005***	Stationary I(2)
Log Rental income Taxes	Ye s	0	-.22 80	0.44 8	Stationary I(2)
Log of GD P	Ye s	0	-4.041	0.007 * **	Stationary I(2)
Lo g of GD Pat factor income	Yes	0	-4.744	0.001***	Stationary I(1)
Log of total Taxes	Yes	0	-6.67 7	0.000	Stationary.1(1)
Log of adjusted direct Taxes	N o	0	-0.13 6	0.931	Non- Stationary.1(2)
Log of adjusted total taxes	Ye s	0	-4.62 6	0.002	Stationary.1(1)

- *** means stationary at 1% level
- I(1) means integrated of order one
- 1(2) mean s integrated of order two
- Time variable: Year, 2007 to 2017.

The PP test confirmed results obtained via ADF test. Hence variables Corporation taxes, direct taxes, Personal income taxes, adjusted direct taxes, Rental income taxes and GDP at factor income in their logarithms were found to be non-stationary even when integrated of order one.

Form the above, the presence of non-stationary variables in all equations to be estimated compels us to use appropriate econometrics technique to deal with that problem. In that sense, the cointegration technique developed by Johansen Cointegration and Engle and Granger (1987) is used in this study. This technique will help us to avoid the problem of spurious regression and to derive the long and short run relationship between variables via error correction modeling.

Cointegration Test

To verify or test for the presence of the long-run relationship or not in the variables under investigation, Johansen cointegration tests were done. The regression residuals were integrated for order zero [1(0)] for stationary. If the time series data are stationary of [1(0)], or there is stochastic trends or variables are individually of [1(1)], any linear combination in the variables will automatically eliminate the stochastic trends resulting to meaningful regression which is not spurious.

Table 8: Results for the Cointegration test.

Eigen value	Likelihood ratio	Critical value of 5%	Critical Value of 1%	Hypothesized No. of CE(S)
0.229	17.601	15.41	20.04	None
0.115	5.625	3.76	6.65	At most 1 **

Note: ** Denotes rejection of the hypothesis at both 1% and 5% level of significance.
L.R. test shows the presence of 2 co-integrating equations at 5% level of significance

The results of table 7 above shows the presence of two statistically cointegrating equations as evidenced by the likelihood ratio greater than the critical value at 5% level of significance. This shows the presence of a linear combination among the variables thus the Granger Causality test was necessary to establish the causation of the linear relationship.

Granger Causality Test

The results of Granger causality test showed that there was a bi-directional a two-way causality from tax revenue to GDP and from GDP to tax revenue in Kenya during the period under investigation. Ordinarily, Granger causality test is conducted to find the causation between the two or more variables. This results was as reported in table 8.

Table 9: Results of Granger causality test

Hypothesis	F-Statistic	Lags	Probability	Conclusion
The Tax does not granger cause GDP	4.627*	4	0.011	Two-way causality
The GDP does not Granger Cause tax	2.752*	4	0.026	Two-way causality

Note: * Denotes rejection of the null hypothesis at 5% level of significance.

The results shows that the granger causality runs from both directions, that is from GDP to tax and from tax to GDP.

Tax Elasticity

This study used GDP as the base for all the sources of the direct revenue tax revenue.

Cointegration analysis for tax elasticity.

All the tests for cointegration depicted existence of long-run relationship between various individual sources of revenue and the national income (GDP).

Table 10: Cointegration Tests for Various Sources of Revenue Corporation Income Tax

Variable (log of Corporation Income Tax) tax)	Lag(s)	ADF	Probability	Decision
Residual (error m)	1	-5.316	0.0000	Stationary/ Co-integrated

Personal Income Tax

Variable (log of Personal Income tax) tax)	Lag(s)	ADF	Probability	Decision
Residual (error m)	1	-5.824	0.0353	Stationary/ Co-integrated

Rental Income Tax

Variable (log of Rental Income Tax) tax)	Lag(s)	ADF	Probability	Decision
Residual (error m)	1	-3.985	0.0122	Stationary/ Co-integrated

Total Direct Income Taxes

Variable (log of Total Direct Income Texes) tax)	Lag(s)	ADF	Probability	Decision
Residual (error m)	1	-4.739	0.0001	Stationary/ Co-integrated

Results reported in Table 9 above indicated that in the long run these sources of revenue were responsive to changes in GDP level. There is cointegration between adjusted Corporation Income tax and GDP at factor income, between adjusted Rental Income tax and GDP at factor income and between adjusted Personal Income tax and GDP at factor income evidenced by a probability less than 0.05.

Table 11: Cointegration Coefficient for Various Sources of Revenue

Variable (Log Corporation Income tax)	Coefficient	t-Statistic	P>/t/	F (1, 10)
Log GDP	3.820	22.09	0.0001	R-Squared = 0.9606
Constant	-94.53	-19.73	0.0001	Adjusted R-Squared = 0.9587

Personal Income tax

Variable (Log Personal Income tax)	Coefficient	t-Statistic	P>/t/	F (1, 10)
Log GDP	5.427	6.40	0.0001	R-Squared = 0.6720
Constant	-136.714	-5.95	0.0001	Adjusted R-Squared = 0.6556

Rental tax

Variable (Log Rental Income tax)	Coefficient	t-Statistic	P>/t/	F (1, 10)
Log GDP	2.326	25.11	0.0001	R-Squared = 0.9693
Constant	-53.600	-20.88	0.0001	Adjusted R-Squared = 0.9677

The coefficient of cointergration results reported in Table 11 above confirmed that there is cointegration between adjusted Corporation Income tax and GDP at factor income, between adjusted Personal Income tax and GDP at factor income, between adjusted Personal Income tax and GDP at factor income was significant with a p-value less than 0.05. For those cointegrating relationship, the research estimated the static equations in order to get the long run elasticities which arc indeed super consistent.

Table 12: Long run elasticities

Coefficient (tax to base)	T Test	F test	R J	D W test	D W test
Corporation Income tax	0.439	0.00 0	102.67	0.85	1.41
Rental Income tax	0.314	0.00 0	82.15	0.8 2	0.6 0
Personal Income tax	0.90 8	0.00 0	475.11	0.9 6	0.58
Total Direct taxes	0.592	0.00 0	360.62	0.9 5	0.9 0

From table 12 above, the elasticity of Corporation Income tax is 0.43, meaning that 1% increase in value of in GDP at factor income lead to a 0.43% increase in Corporation taxes in the long run. Rental income taxes have a long run elasticity of 0.31, implying that a 1% increase in GDP at factor income raise rental taxes by 0.31%. The long run elasticity of personal income tax is comparatively higher as in the long run. 1% increase in private final consumption lead to a 0.90% increase in personal income taxes. Finally, the total direct taxes elasticity is 0.59; in other words a 1% increase in GDP lead to 0.59% increase in total taxes. As a matter of fact, it should be noticed that these rates of change concern the taxes adjusted to isolate effect of discretionary changes. Coefficients in all equations are significant at 1% level of significance with high F test value for overall significance and R squared. However Durbin Watson values are lower but given that the coefficients are super consistent, this should not be a problem. Given the above cointegration relationship, short ran relationship can be obtained via error correction modeling. The following table 12 report results for short term elasticities.

Error Correction Model (ECM) Analysis- Elasticity

The ECM links both the short-run and the long run dynamics of the model. The ECM was developed by running a regression of the non-stationary endogenous

variable against non-stationary exogenous variables and the error correction term (ECT). The results are shown in table 11 below.

Table 13: The ECM analysis- Elasticity Corporation Income Tax

L log Income tax	Coefficient	t- Statistic	p>/t/	F (2, 10) = 0.63
L1 Log GDP	0.1978	0.15	0.883	Prob > F = 0.5460
L error m	0.3201	1.63	0.121	R^2 = 0.0827
Constant	0.1285	2.59	0.019	Adjusted R^2 = 0.0483

Personal Income Tax

L log Income tax	Coefficient	t- Statistic	p>/t/	F (2, 10) = 1.35
L1 Log GDP	0.7454	0.83	0.418	Prob > F = 0.2860
L error m	0.5070	1.07	0.303	R^2- = 0.1369
Constant	-0.1008	-0.24	0.812	Adjusted R^2 = 0.0354

Rental Income Tax

L log Income tax	Coefficient	t- Statistic	p>/t/	F (2, 10) = 2.05
L1 Log GDP	0.3503	-0.84	0.416	Prob > F = 0.1629
L error m	0.1391	0.44	0.665	R- Squared = 0.2149
Constant	0.3146	1.70	0.109	Adjusted R- Squared = 0.1103

Total Direct Income Taxes

L log Income tax	Coefficient	t- Statistic	p>/t/	F (2, 17) = 3.83
L1 Log GDP	0.7778	0.25	0.083	Prob > F = 0.0001
L error m	0.1302	1.43	0.011	R- Squared = 0.8204
Constant	0.1285	2.89	0.019	Adjusted R- Squared = 0.7993

The ECM analysis, illustrated in Table 13, showed that in the short- run none of the explanatory variables in the model had a significant impact on the explained variable. This shows that in the short-run the direct taxes (Corporate Income tax, Personal Income tax and the rental Income tax) not elastic in the short- run.

Interpretation of the Findings

The result of ADF test and PP the unit root test shows that all the variables were non-stationary at level. The null hypothesis was therefore rejected since the test statistic values for both ADF and PP were all greater than the critical values at 10% level of significance, thus the unit root for all in the variables was present. This shows an existence of a unique long-run relationship among the variables in the model. Thus precise interpretation in economic terms of the long-run relationship between various variables in the models can only be achieved through normalization of the unconstrained cointegration equations estimates.

Through normalization, the results showed elasticity coefficient of 0.592 for Kenya's direct taxes. This clear means that a positive change in national income spurred a less than proportionate change in tax revenue since the elasticity coefficient is less than a unit hence inelastic. This findings are consistent with that of other researchers such as the findings of Moyi and Muriithi (2003) [14] and the finding of Wawire (2006) [29] who concluded in there study that the overall tax system in Kenya is inelastic. However, the results deviate from the findings of both Wang'ombe (1999) [28] and Waris *et al.* (2009) [27] who found an elasticity coefficient of 1.27 and 1.69 respectively. Wang'ombe (1999) [28] findings was however old and the environment have changed tremendously while Waris *et al.* (2009) [27] falls within the period of study for this paper.

In the long-run, the coefficient of error correction shows the speed of adjustment of variables towards equilibrium value. The direction of adjustment towards equilibrium is shown by the sign of the coefficient of error correction thus the higher the coefficient, the faster the speed of adjustment towards equilibrium level. Further, a negative sign of the coefficient implies convergence towards the equilibrium in the long-run. The results indicate that the error correction coefficient for the direct tax in Kenya is 0.7778 which was significant. This shows a high speed of adjustment, meaning that 77.78 percent of the errors in the previous tax revenue are corrected in the current period.

As indicated in table 13, the error correction coefficient for corporation taxes was 0.1978 which was also statistically significant, meaning a low speed of adjustment. This means that 19.78 percent of the errors in the previous tax revenue are corrected in the current period. The log coefficient for GDP was significant with a t-ratio of 17.601. The coefficient of the log GDP was positive implying that a 1% GDP increases leads to total tax revenue to increase by 0.1285%. This means that the tax system yielded 0.1285% change in tax revenue as a result of changes in discretionary measures for every 1% change in GDP.

If we look at total direct taxes in general, the elasticity is 0.592. As in the case of various taxes taken individually, the tax system is not productive in general. Moreover, the difference in elasticity between individual taxes forming direct taxes highlights how important and vital, discretionary tax measures and reforms have been to the Kenyan tax system. Thus we can conclude that reforms have improved the productivity of taxes in Kenya.

Conclusion

In conclusion, this paper has revealed that the productivity of Kenyan system of direct taxes is still low with an elasticity coefficient less than a unit (0.592), hence the system of direct taxes inelastic in the short run. There is a need take into account the economic structure of the tax

system and its capacity to efficiently administer taxes. With an error correction coefficient of 0.7778, we can conclude that the direct tax revenue in Kenya is actually not responsive enough to changes in income growth since the coefficient of elasticity was less than a unity. However, the continuous growth in revenue collection was due to changes in budgetary that have increased the responsiveness of tax revenues to changes in national income. The discretionary tax measures have had an overall impact on growth in total revenue over the period. However, the pure responsiveness of tax revenue to changes in national income was found to be less than unity. Thus the system of direct taxes is not productive in general.

The findings of this paper established that a positive change in national income spurred a less than proportionate change in tax revenue since the elasticity coefficient is less than a unit hence inelastic. However, high speed of adjustment towards tax equilibrium was very high shown by a error correction coefficient of 0.7778 which was statistically significant. This means that 77.78 percent of the errors in the previous tax revenue are corrected in the current period.

With a positive log coefficient for GDP statistically significant with a t-ratio of 17.601, a 1% GDP increases leads to total tax revenue to increase by 0.1285%. This means that the tax system yielded 0.1285% change in tax revenue as a result of changes in discretionary measures for every 1% change in GDP. If we expand the tax base and net more sector of economy that contribute to GDP but does not contribute significant tax revenue such as the informal sector and the extractive sector, the tax revenue productivity will increase and the change in revenue will be more proportionate to changes in GDP.

Measures such as provision of tax exemptions and incentives which are not revenue neutral, low compliance in the informal sector whose activities is not readily observable can partly explain the low elasticity of tax observed in the direct tax system in Kenya. Low economic cycles may shift a business from formal to informal sector which leads to revenue loss.

Recommendations

Based on these findings, we strongly recommend that the government to re-evaluate the tax modernization issues so as to fill the gaps of inefficiency by identifying new opportunities for mobilizing revenue collection through expansion of the catchment area. The growth rate of government expenditure adjusted for discretionary tax measures revenue should be less than near term GDP growth. A scientific method of determining targets for revenue administrations will help to reduce the proportion of actual collection attributed to discretionary measures and focus on pure responsiveness of revenue to national income changes.

This paper also strongly recommend urgent redefinition and formalizing of the information sector widen the tax base in Kenya. The informal sector covers a large spectrum of economic activities ranging from commerce, agriculture, construction, manufacturing, transportation and services that are not captured in the tax bracket in Kenya. The sector is not structured in its approach to business engagement, but represents 82.7% of the workforce in Kenya (majority of who are youth) and contributes more than 40% of the GDP yet little to none tax revenue is collected from the sector. Thus urgent redefinition of this sector through a deliberate effort to re-profile the economic activities in the sector by empowering those in the sector with basic but important business best practice such as keeping records, complying with tax law and other beneficial statutory requirements such as health insurance (NHIF) and social security (NSSF) to achieve the goal of formalizing and monitoring the volume of the transaction in the sector. In addition, government can increase informal sector tax base by formalizing core value chains in economic activities. Partnership between International Labor Organization and governments has helped to formalize value chains through supporting functions like infrastructure and rules and regulations including legislation. Revenue bodies will better understand the entrepreneurial ecosystem elements of the once informal sector's framework (e.g. business infrastructure) and systematic conditions (e.g. Networks and finance) and design interventions to facilitate tax assessment and collection.

Finally in an effort to expand the tax base and improve the productivity of the tax system, the government should to develop the policy, legal and institutional framework for the exploitation and taxation of Kenya's natural resources in the extractive sector. The extractive sector in Kenya and all of Africa countries has the potential to generate a significant amount of the much needed tax revenues to finance the socio-economic development. In Kenya there has recent discovery of oil in Turkana County in 2012, natural gas in Lamu, gold in Kakamega and Migori and coal in Kitui, thus Kenya Revenue Authority (KRA) should focus on formalisation/streamlining of the artisanal mining sector to allow for fair taxation of this category while encouraging fair compensation or employment for small scale miners.

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